The Beta Anomaly within the S&P 500 Stocks Universe

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Motivation: The Beta Anomaly

- Efficient markets are characterized by the fundamental risk-return relationship:

Above-average returns are realized by taking higher risks

- Contemporary research has put this notion into the test implying another risk-pricing anomaly and another puzzle (Baker et. al., 2011; Frazzini and Pedersen, 2014; Auer and Schuchmacher, 2015; Ang et. al., 2009; Li et. al., 2014)
- This *inverse risk-return relationship* is referred to in the literature as the:

'low-risk anomaly' or 'low-volatility anomaly'

Low-volatility can refer to: (1) total volatility, (2) idiosyncratic volatility, (3) beta

Motivation: The Beta Anomaly

Causes, origins & implications

Causes

- Irrational demand for risk (i.e. overinvestment in high-beta stocks)
- 'Irrational' means 'not explained by fundamentals'
- Why there is irrational demand for high-risk assets?
  - Black (1972) & Frazzini-Pedersen (2014): Investors face funding constraints
    More on 'Theoretical Model'
- 'Limits of Arbitrage' literature: limited capital provision and other costs faced *by arbitrageurs* in the process of eliminating mispricings → Gromb and Vayanos (2010)

Implications

- Underlying portfolios of assets inefficiently priced:
  - Low-beta assets are underpriced
  - High-beta assets are overpriced (possibly)

Declining $\hat{\alpha}_p$ pattern in beta-sorted assets, but not necessarily $\hat{\alpha}_p < 0$

- *High-beta* must be associated with *low-alpha,* and *low-beta* with *high alpha.*
- Flatter Security Market Line (SML) than Standard CAPM → A result that goes back to BJS (1972)
- An 'arbitrage' opportunity: Long low-beta assets & short high-beta assets

→ … or other investment opportunities:
possibly exploit the lower-risk securities if only interested in a long position

- The Literature -

Agent types:
- arbitrageurs
- outside investors

Model Phenomena:
- contagion
- amplification

Motivation: This study

Contribution to Literature

- Anomalies become weaker and arbitrage strategies unprofitable over time and once published (e.g. size effect 1980's, BKM p.373). Beta-effect still holds?
  - Consider both entire period (1926-2017) and a more recent period (2009-2017)
  - Incorporate newly introduced factor portfolios (FF5; 2015) to test whether they explain the remaining component (i.e. alpha) of excess returns related to the beta anomaly
- Robustness of results in alternative stocks' space (i.e. more conservative inclusion of stocks) and betas?

Motivation: This study

Contribution to Literature

- Persistence of anomalous returns due to illiquid and smaller stocks? → implying 'transaction costs' as the source of persistency
  - Views on 'illiquidity concerns' in the literature:

    Li et. al. (2014)

    Long-short strategy for *idiosyncratic volatility anomaly* for entire CRSP stocks universe is profitable, but becomes unprofitable when small and low-liquidity stocks are omitted (1963-2010)

    Auer and Schuchmacher (2015)

    Long-short strategy for *beta anomaly* among DJIA constituents is profitable (1926-2013)
  - Embed illiquidity concerns by considering the actively traded (i.e. liquid), large and seemingly efficient S&P 500 constituents universe
  - Evidence on practical implementation *of long-short arbitrage strategies* that exploit the difference between low and high beta stocks and *long-only strategies* that exploit the mispriced decile portfolios

Need more motivation to study the beta anomaly?

More Motivation:

… has initiated a new trend in the passive funds industry

- Monthly net inflows ($bn) in funds that provide exposure to low-volatility equities

Source: FT.com

Just as we were talking about arbitrage strategies becoming un-profitable…
ETF's are usually only 'long' though

Theoretical Model

Explaining the Beta Anomaly

- Black (1972) and Frazzini-Pedersen's (2014) Margin CAPM:

  Extend the standard CAPM to include *leverage constraints*

- Idea:
  Constrained investors buy riskier (high-beta) assets, in order to compensate for insufficient leverage and taking advantage of the safer (low-beta) stocks.
  - Constrained investors buy riskier assets
  - Unconstrained investors buy safer assets and leverage

- Who is constrained?

  Retail investors and **professional investors** legally not allowed, or restricted to use leverage (e.g. mutual funds). Baker et. al. (2011) point to *benchmarking*.

- Equilibrium required return for asset s, when investors face leverage constraints:

$$E_t(r_{t+1}^s) = r^f + \underbrace{\psi_t(1-\beta_t^s)}_{\substack{\text{Jensen's alpha} \\ \text{(higher beta implies lower alpha)}}} + \beta_t^s(E_t\,(r_{t+1}^M) - r^f) \quad \leftarrow \text{ Margin CAPM}$$

A word on the SML

Empirical Findings Suggestive of the Beta Anomaly

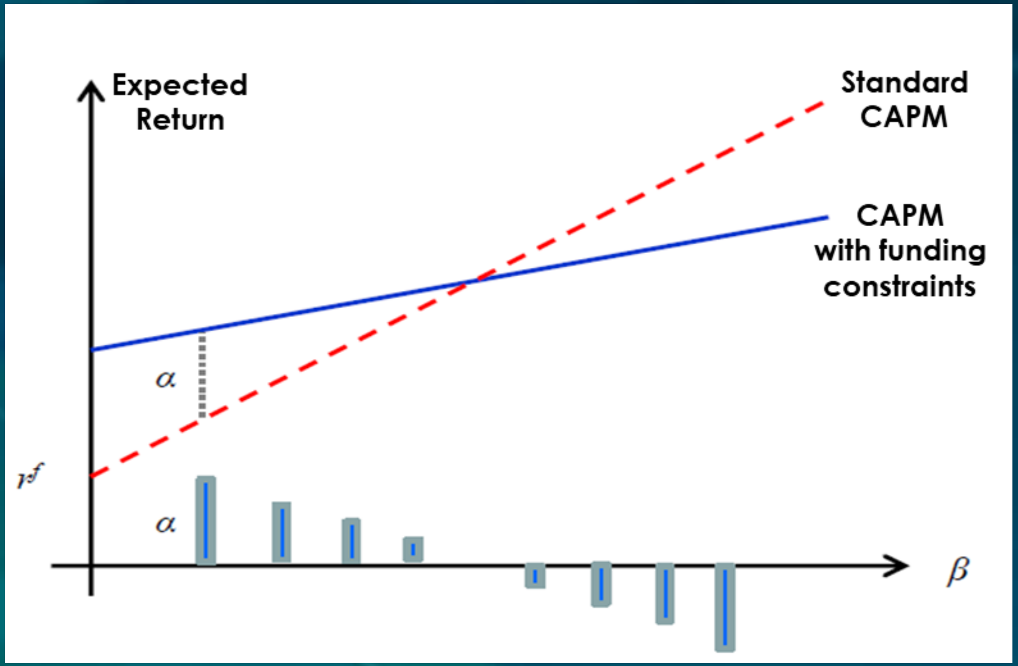
- Empirical SML estimation via the 2nd pass (cross-section) regression:

$$\overline{r_i - r_f} = \gamma + \delta \cdot \beta_i + \epsilon_i$$

where β_i is the **estimated CAPM beta** from the 1st pass (timeseries) regression for stock/portfolio i

- Estimated δ smaller than standard asset-pricing theory predicts. Empirical findings on the flatness of SML go back to:
  - Lintner (1965) on individual stocks
  - Black, Jensen, and Scholes (1972) on portfolios

- Funding constraints provide a potential explanation to the **empirically observed** flatter SML, as the underlying **theoretical model** supports a flatter SML



Not necessarily negative alpha

Research Design: Coverage & Sources

- Coverage:
  - January 1926 – December 2017 (Entire) | March 1964 – December 2017 (S&P 500)
  - All common U.S. equities (CRSP share code 10 or 11) ← Referring to as 'Entire Universe of Stocks'
- Daily stock returns, adjusted for stock splits and dividends, from Center for Research in Security Prices (CRSP)
- Factor portfolios and risk-free rate from WRDS section 'Fama-French Portfolios and Liquidity Factors' and Kenneth French's website
- Market return is calculated based on the CRSP value-weighted market index which includes all non-ADR securities listed in NYSE, AMEX and NASDAQ exchanges
- Excess returns calculated with respect to the one-month US Treasury bill rate
- Historical constituents for S&P 500 Composite Index from Compustat-Capital IQ 'Index Constituents'. Establish a link between Compustat and CRSP using CUSIP identifier.
- The S&P 500 with dividends employed as the passive investment portfolio for mixed-portfolios and as the benchmark asset in Sharpe Ratio comparisons, was retrieved from CRSP's 'S&P 500 Indexes' file

- **Equities**
24.698
- **Days**
23.005
- **Month-Ends**
1.052

Research Design: Empirical Model

Beta-Decile Portfolios

STEP 1: Assess the existence of the beta anomaly

- **Constructing Quantile Portfolios**

Securities are ranked in ascending order with respect to their estimated ex-ante beta and assigned to one of the ten deciles, and equally weighted portfolios within each decile are formed. Rebalancing takes place every end of month.

- Beta deciles are calculated based on NYSE breakpoints to avoid break in the timeseries

Research Design: Empirical Model

Betting-against-beta Portfolio

STEP 2: Assess the <u>potential profitability</u> of the BAB arbitrage strategy

→ as means of comparing the magnitude of the asset pricing effect between different times and market segments

- **Constructing the betting-against-beta (BAB) portfolio**

Ranked securities are assigned to one of two portfolios: low-beta (assets below median) and high-beta (assets above median), and securities are <u>weighted by their ranked betas</u> within each portfolio.

Lowest $\hat{\beta}_{i,t}$ | Median $\hat{\beta}_{i,t}$ | Highest $\hat{\beta}_{i,t}$

Low-beta portfolio | High-beta portfolio

The BAB is the self-financing zero-beta portfolio that is long the (levered) low-beta portfolio and that short-sells the (de-levered) high-beta portfolio:

$$r_{t+1}^{BAB} = \frac{1}{\beta_t^L}\left(r_{t+1}^L - r^f\right) - \frac{1}{\beta_t^H}\left(r_{t+1}^H - r^f\right)$$

← Offsetting positions in the risk-free asset to make the portfolio **self-financing**

By construction, $\beta^{BAB} = 0$
(i.e. BAB is a **market-neutral** portfolio, but is still exposed to the other systematic risk factors)

Research Design: Empirical Model

Estimation of Pre-ranking Betas

- Ex-ante beta estimator, for stock i: $\hat{\beta}_i^{ts} = \hat{\rho}_{i,m} \frac{\hat{\sigma}_i}{\hat{\sigma}_m}$
  - **Volatilities** estimates:
    1-year rolling standard deviation on one-day log-returns
  - **Correlation** estimates:
    5-year rolling on overlapping 3-day log returns $r_{i,t}^{3day} = \sum_{k=0}^{2} \ln(1 + r_{i,t+k})$, to account for non-synchronous trading (i.e. the fact that trading takes place in non-equidistant time points)
  - Similar findings, using CAPM betas estimated on 252-days of log-returns from month-end

t denotes:
2 days prior to month-end, so that correlations are calculated based on 'past' data only

- To remove noise and outliers, because betas are subject to selection bias, we employ the Vasicek (1973) approach and shrink betas towards one:

$$\hat{\beta}_i^{s} = 0.6 \cdot \hat{\beta}_i^{ts} + 0.4 \cdot 1$$

  - The ranking of sorted stocks is not affected
  - Stocks having a return timeseries with a minimum of 3-year non-missing obs in the past 5 years from month-end AND a minimum of 6 months of non-missing obs in the past 252 days from month-end, are only included (see method A3 in robustness section)

Research Design: Rebalancing Statistics

- Portfolio Rebalancing takes place:
  - 1052 times for the Entire Universe (1st: 30/04/1930, Last: 30/11/2017)
  - 609 times for the S&P 500 Universe (1st: 31/3/1967, Last: 30/11/2017)

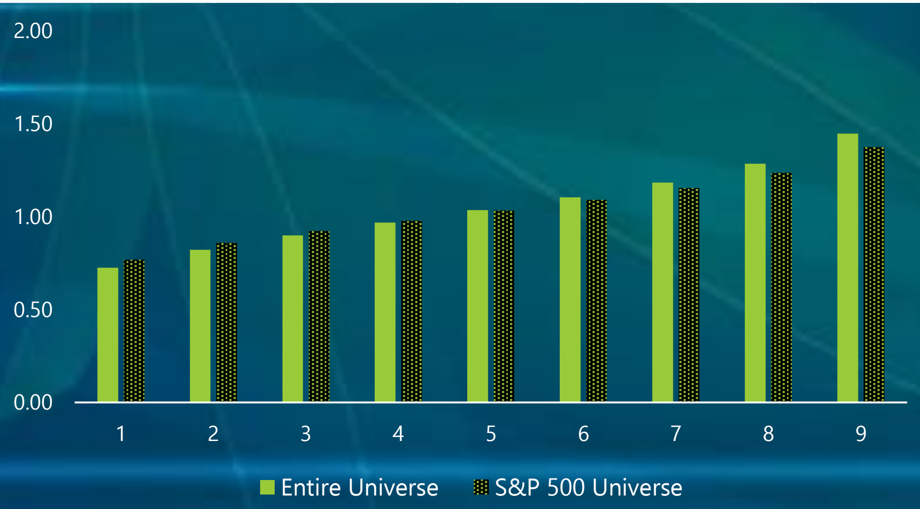
Universe	End-of-Month	Total Securities (All Exch.)	Valid Securities (All Exch.)	Calculated Betas (NYSE)
Entire Universe	19300430	718	172	172
	20171130	3614	2998	1134
	Average	**4881**	**3369**	**1246**
S&P 500 Universe	19670331	338	282	282
	20171130	441	373	285
	Average	**427**	**362**	**331**

Research Design: Rebalancing Statistics

Beta Deciles

- Estimated Beta-Deciles using shrank betas for NYSE stocks (averages for 1967-2017)

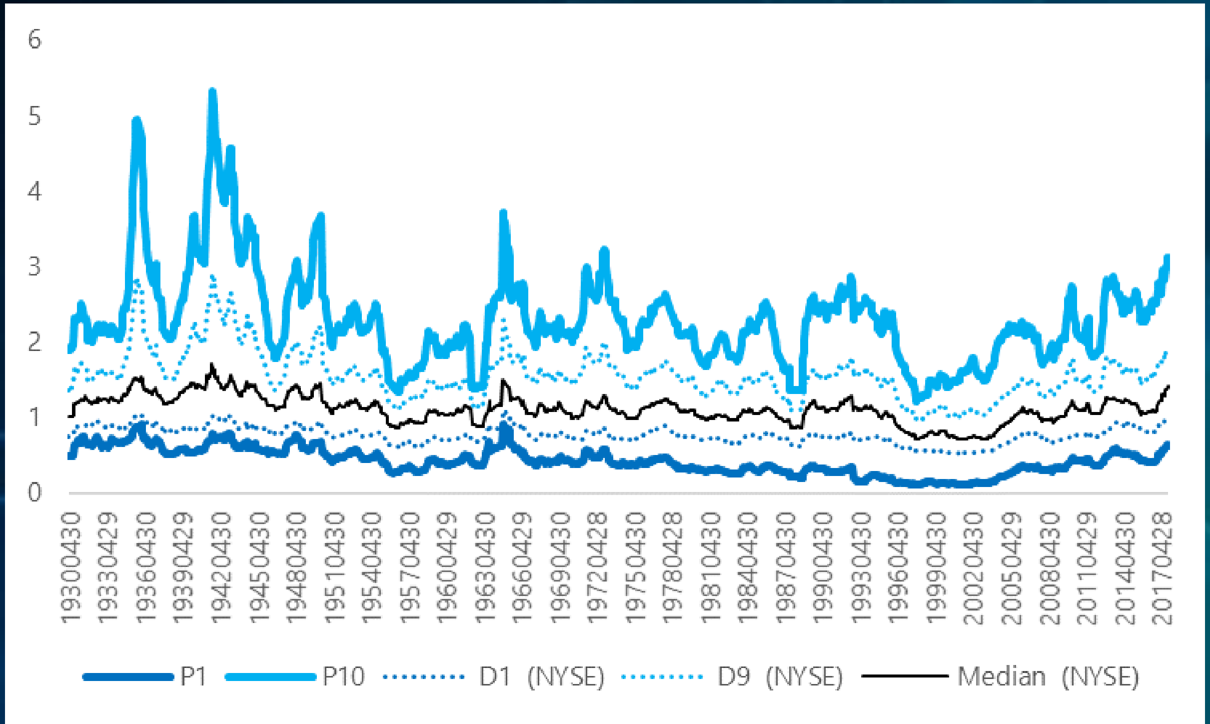
Universe	End-of-Month	D1 (NYSE)	D2 (NYSE)	D3 (NYSE)	D4 (NYSE)	Median (NYSE)	D6 (NYSE)	D7 (NYSE)	D8 (NYSE)	D9 (NYSE)	Median (All Exch.)
Entire Universe	19300430	0.74	0.83	0.90	0.95	1.01	1.09	1.14	1.24	1.35	1.01
	20171130	1.01	1.13	1.23	1.33	1.41	1.49	1.58	1.69	1.92	1.37
	Average	**0.72**	**0.82**	**0.90**	**0.97**	**1.03**	**1.10**	**1.18**	**1.28**	**1.45**	**0.98**
S&P 500 Universe	19670331	0.69	0.77	0.83	0.89	0.96	1.02	1.09	1.20	1.40	0.96
	20171130	0.86	0.99	1.08	1.14	1.23	1.31	1.38	1.46	1.58	1.27
	Average	**0.77**	**0.86**	**0.92**	**0.98**	**1.03**	**1.09**	**1.15**	**1.24**	**1.37**	**1.04**



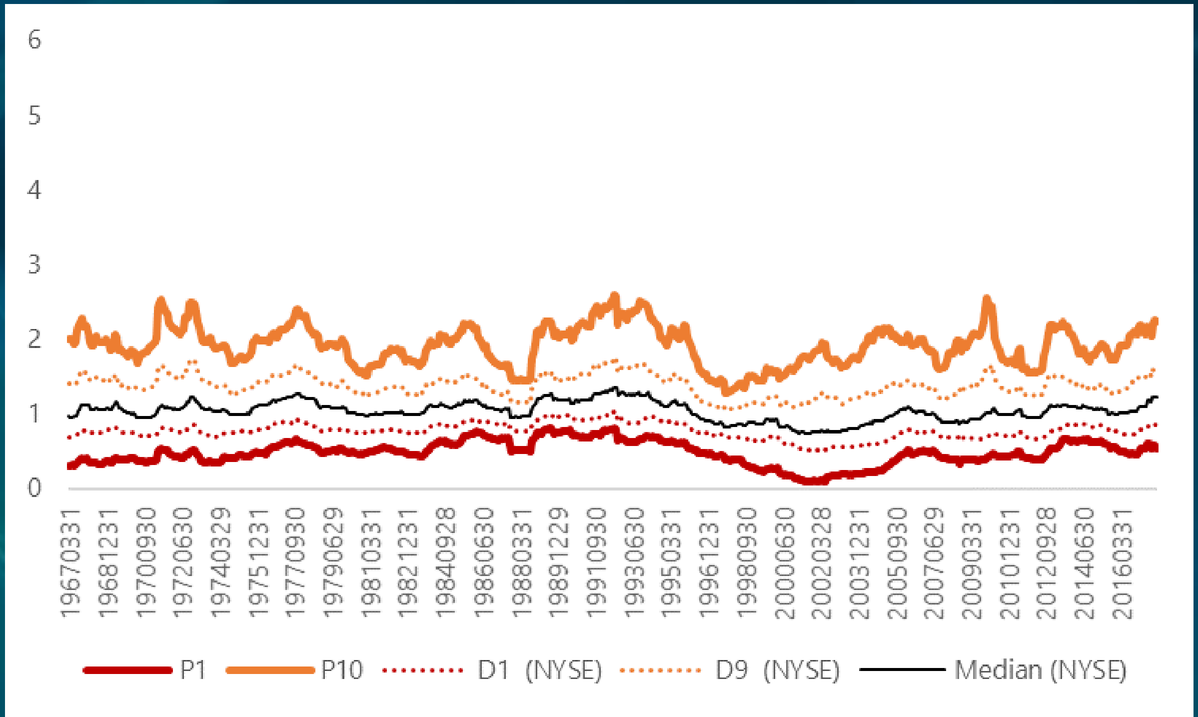
Research Design: Rebalancing Statistics

Beta Deciles and Decile-Portfolio Betas Time-series

Entire Universe (1930-2017)



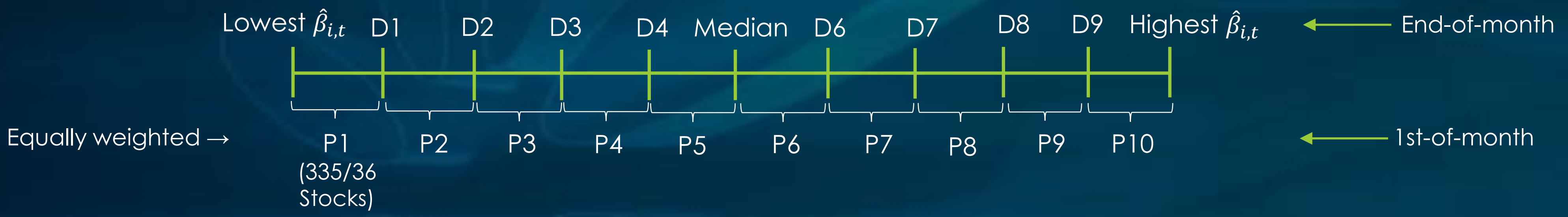
S&P 500 Universe (1967-2017)



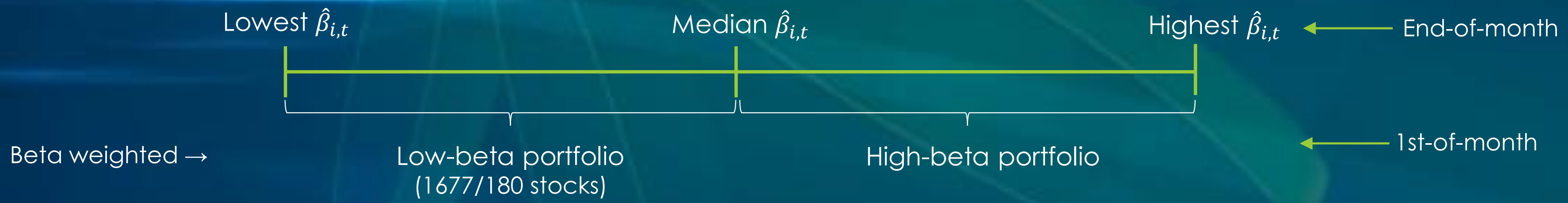
Ex-ante betas of decile-portfolios (P1 to P10) are estimated as the average calculated betas at month-end
Calculating portfolio beta as the weighted average of individual assets' betas is a convention, as betas were not calculated using the Single-Index model

Research Design: Empirical Model

STEP 1: Asses the existence of the beta anomaly



STEP 2: Asses the potential profitability of the BAB arbitrage strategy



The Betting-against-beta Portfolio

Closer Look: Entire Universe

- The self-financing BAB portfolio and its 2 constituents:
  - Low-beta portfolio:
    - The non-rescaled portfolio will have a beta of $\sum_{i=1}^{l} \hat{\beta}_{i,t} w_L^i$ = 0.76
    - Divide by 0.76 to rescale portfolio to have a beta of 1
    - Rescaled (levered) portfolio: Long \$1.3 of low-beta stocks and finance this by short-selling \$1.3 of risk-free securities (Treasury bill)
  - High-beta portfolio:
    - The non-rescaled portfolio will have a beta of $\sum_{j=1}^{h} \hat{\beta}_{j,t} w_H^j$ = 1.48
    - Portfolio is rescaled (i.e. divide by 1.48) to have a beta of 1
    - Rescaled (de-levered) portfolio: Shortsell \$0.7 of high-beta stocks, with \$0.7 earning the risk-free rate

$$r_{t+1}^{BAB} = \overbrace{\underbrace{\boxed{\frac{1}{\sum_{i=1}^{l} \hat{\beta}_{i,t} w_L^i}}}_{1/0.76\,=\,1.35} \left(\sum_{i=1}^{l} r_{t+1}^i w_L^i - r^f\right)}^{\text{Low-beta levered portfolio}} - \overbrace{\underbrace{\boxed{\frac{1}{\sum_{j=1}^{h} \hat{\beta}_{j,t} w_H^j}}}_{1/1.48\,=\,0.7} \left(\sum_{j=1}^{h} r_{t+1}^j w_H^j - r^f\right)}^{\text{High-beta de-levered portfolio}}$$

- By construction $\sum_{i=1}^{l} w_L^i = 1$ and $\sum_{j=1}^{h} w_H^j = 1$. The weight for each security i (with return r_{t+1}^i) in the low-beta portfolio will be $\frac{1}{\sum_{i=1}^{l} \hat{\beta}_{i,t} w_L^i}\left(\sum_{i=1}^{l} w_L^i\right)$, and similarly for each security j in the high-beta portfolio. Therefore, both the Low-beta levered and the High-beta de-levered portfolios will have betas of 1. This results in $\beta^{BAB} = 0$ (i.e. market neutral).

Research Design: Empirical Model

Evaluation of Portfolios

STEP 3: Assessing Constructed Portfolios' Performance

Two measures of *risk-adjusted returns*:

(1) Sharpe ratios: Excess return per unit of risk

(2) Alphas: The constant in factor regressions that extend the standard CAPM to account for the primary asset-pricing anomalies

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_{p,1}(r_{m,t} - r_{f,t}) + \beta_{p,2}SMB_t + \beta_{p,3}HML_t + \beta_{p,4}UMD_t + \beta_{p,5}LIQ_t + \epsilon_{p,t}$$

$$r_{p,t} - r_{f,t} = \alpha_p + \beta_{p,1}(r_{m,t} - r_{f,t}) + \beta_{p,2}SMB_t + \beta_{p,3}HML_t + \beta_{p,4}RMW_t + \beta_{p,5}CMA_t + \epsilon_{p,t}$$

Statistically significant alphas would suggest excess returns in each portfolio p, that cannot be explained by the standard risk factors.

Inferencing based on risk-adjusted returns:

- Decile portfolios: Declining S.R./$\hat{\alpha}_p$ pattern in $\hat{\beta}_p^s$ sorted portfolios ⇒ evidence that beta-anomaly exists
- BAB portfolio: Capturing the magnitude of the asset pricing effect (i.e. 'arbitrage' strategy profitability)

Research Design: Empirical Model

Asset-Pricing Models Literature

Fama/French (1993)

- **Size:** Small company stocks outperform large
  - Measured as: small-minus-big (SMB)
- **Book-to-market:** 'Value' stocks (high BE/ME) outperform 'growth' stocks (low BE/ME)
  - Measured as: high-minus-low (HML)

Carhart (1997)

- **Momentum:** Stocks that have been doing well tend to outperform those that have been doing poorly
  - Measured as: up-minus-down (UMD)

Pastor/ Stambaugh (2003)

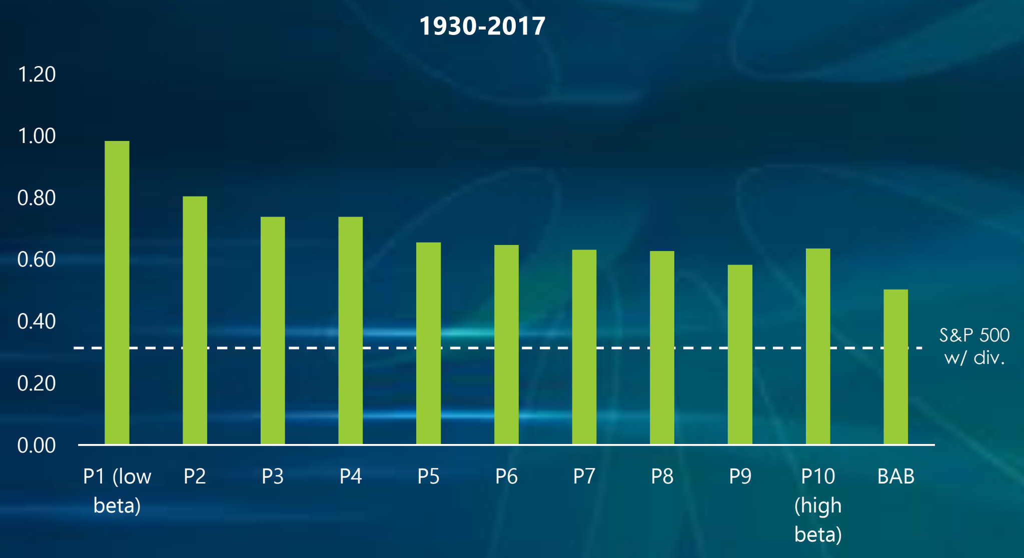
- **Liquidity:** Stocks sensitive to liquidity shocks outperform those that are less sensitive
  - Measured as: traded liquidity factor (LIQ)

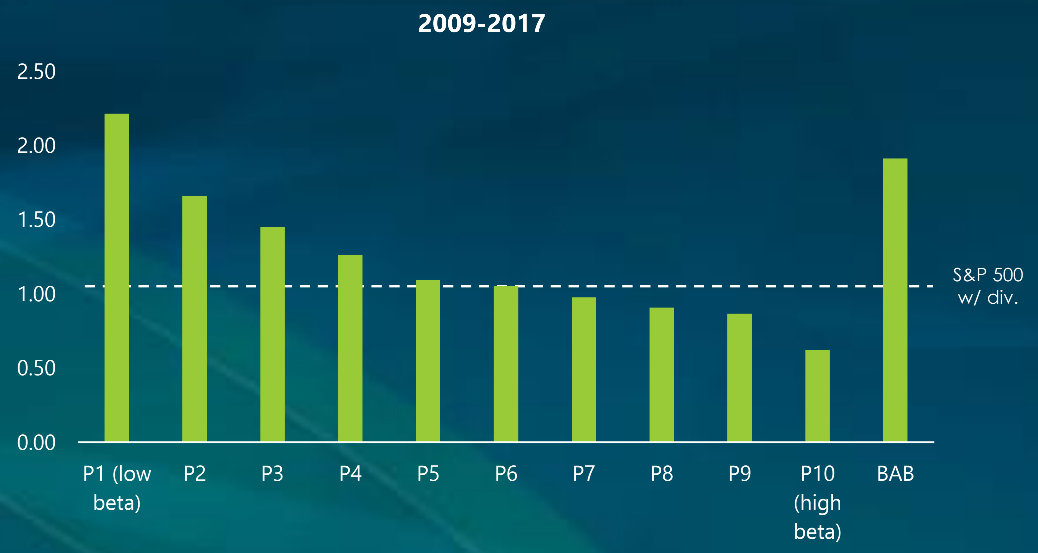
Fama/French (2015)

- **Profitability:** Stocks of highly profitable companies outperform those of less profitable companies
  - Measured as: robust-minus-weak (RMW)
- **Investment patterns:** Firms with conservative investment portfolio outperform those with aggressive investment portfolio
  - Measured as: conservative-minus-aggressive (CMA)

Results: Performance Evaluation

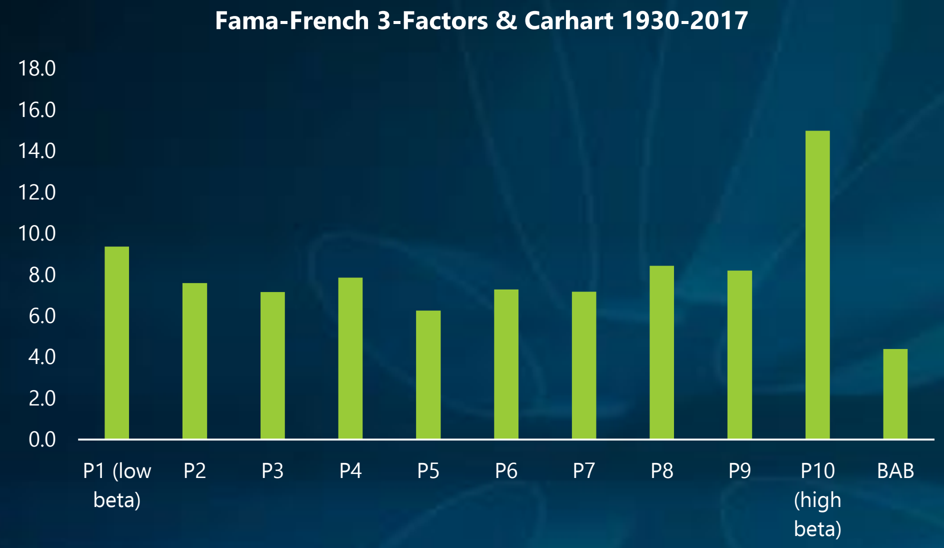
Sharpe Ratios, Entire Universe

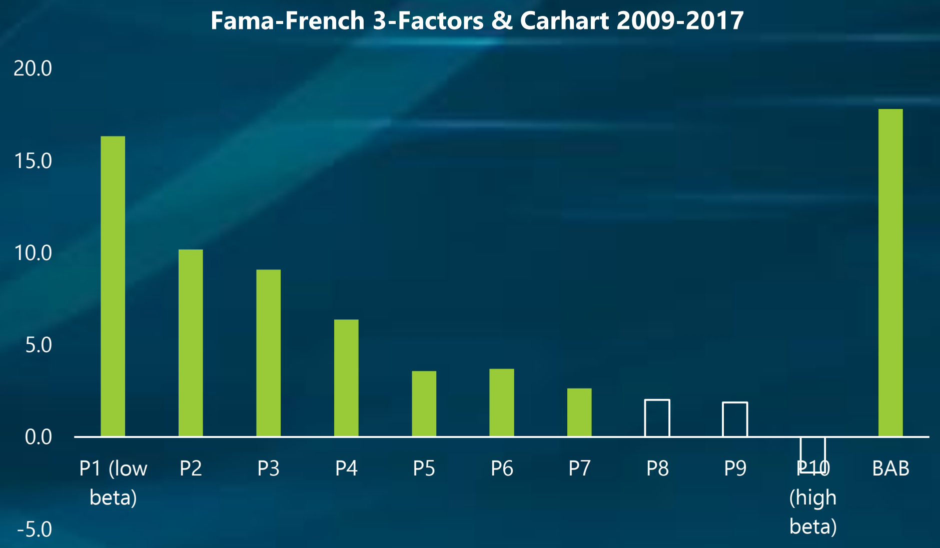


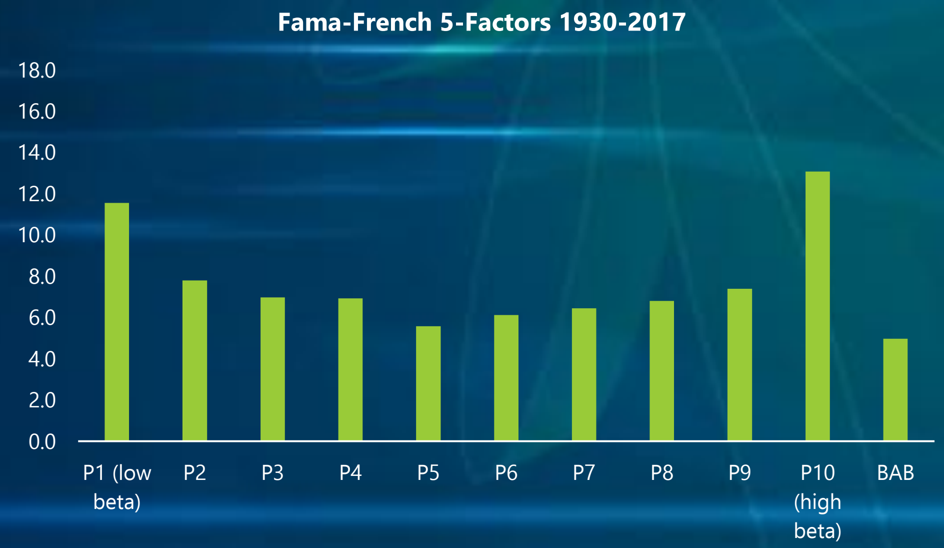


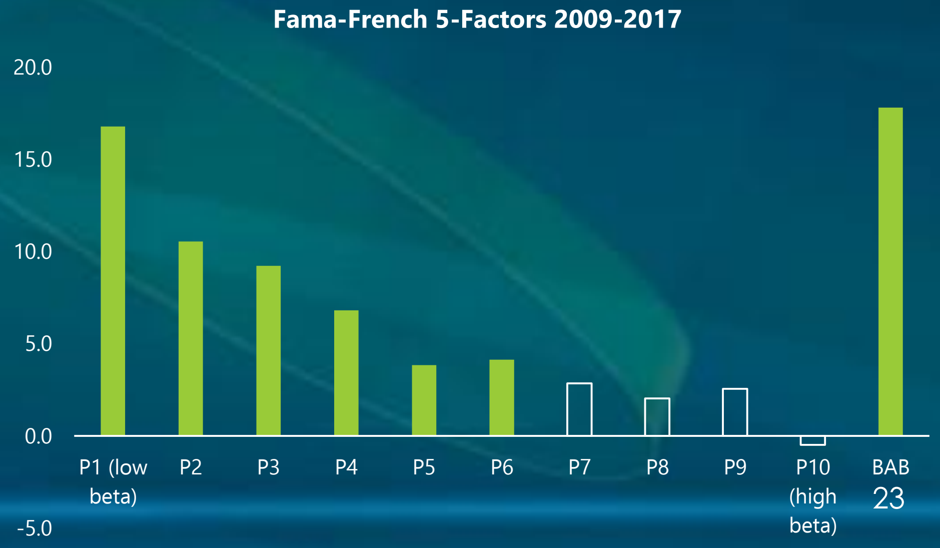
Results: Performance Evaluation

Alphas, Entire Universe









Results: Performance Evaluation
Entire Universe

Portfolio	P1 (low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)	BAB	S&P 500 (w/div.)
					1930-2017							
alpha 4-factor	**9.4**	**7.6**	**7.2**	**7.8**	**6.2**	**7.3**	**7.2**	**8.4**	**8.2**	**15.0**	**4.4**	
p-Value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
alpha FF5-factor	**11.6**	**7.8**	**7.0**	**6.9**	**5.6**	**6.1**	**6.4**	**6.8**	**7.4**	**13.1**	**5.0**	
p-Value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	
Excess Return	15.0	14.2	15.1	17.2	16.4	18.1	18.1	19.8	20.4	28.2	5.7	5.7
Volatility	15.3	17.7	20.4	23.3	25.0	27.9	28.7	31.5	34.9	44.4	11.3	18.7
Sharpe Ratio	0.98	0.80	0.74	0.74	0.66	0.65	0.63	0.63	0.58	0.64	0.50	0.31
beta CAPM	0.69	0.86	1.02	1.17	1.25	1.38	1.43	1.54	1.70	1.99	-0.07	
beta 4-factor	0.62	0.76	0.89	1.02	1.08	1.16	1.20	1.28	1.40	1.51	-0.01	
					2009-2017							
alpha 4-factor	**16.3**	**10.2**	**9.1**	**6.4**	**3.6**	**3.7**	**2.6**	2.0	1.9	-1.9	**17.8**	
p-Value	0.00	0.00	0.00	0.00	0.00	0.01	0.09	0.25	0.41	0.68	0.00	
alpha FF5-factor	**16.8**	**10.5**	**9.2**	**6.8**	**3.8**	**4.1**	2.8	2.0	2.5	-0.5	**17.8**	
p-Value	0.00	0.00	0.00	0.00	0.01	0.03	0.16	0.46	0.48	0.94	0.00	
Excess Return	25.50	22.48	23.32	22.36	20.56	22.02	22.02	23.45	24.95	24.79	16.09	14.0
Volatility	11.51	13.56	16.07	17.68	18.82	20.94	22.55	25.81	28.77	39.87	8.41	13.27
Sharpe Ratio	2.21	1.66	1.45	1.26	1.09	1.05	0.98	0.91	0.87	0.62	1.91	1.06
beta CAPM	0.67	0.87	1.04	1.17	1.27	1.39	1.49	1.68	1.83	2.33	-0.24	
beta 4-factor	0.52	0.72	0.82	0.94	1.01	1.07	1.12	1.21	1.28	1.43	-0.07	

Based on regressions using monthly frequency simple returns. All figures are annualized.

Implied Empirical SML versus CAPM SML

2009-2017, Entire Universe

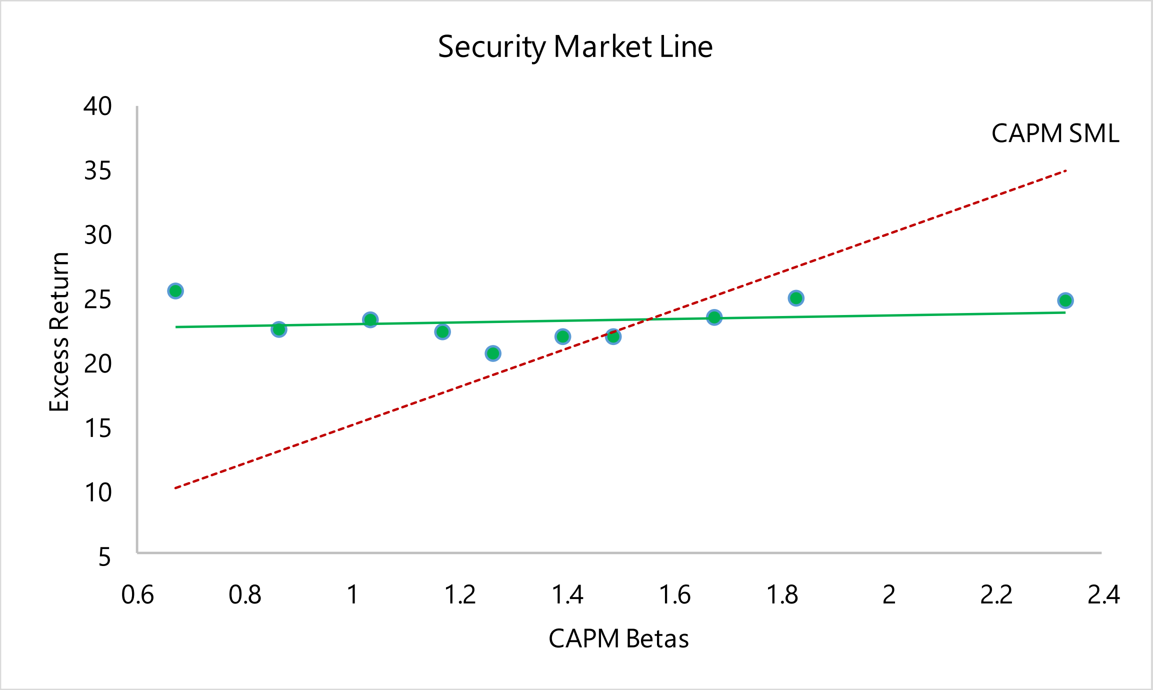
- The presented findings suggest that the SML has continued to be flatter than CAPM predicts

- Empirical SML estimation via the 2nd pass (cross-section) regression following Black, Jensen, and Scholes (1972):

$$\overline{r_i - r_f} = \gamma + \delta \cdot \beta_i + \epsilon_i$$

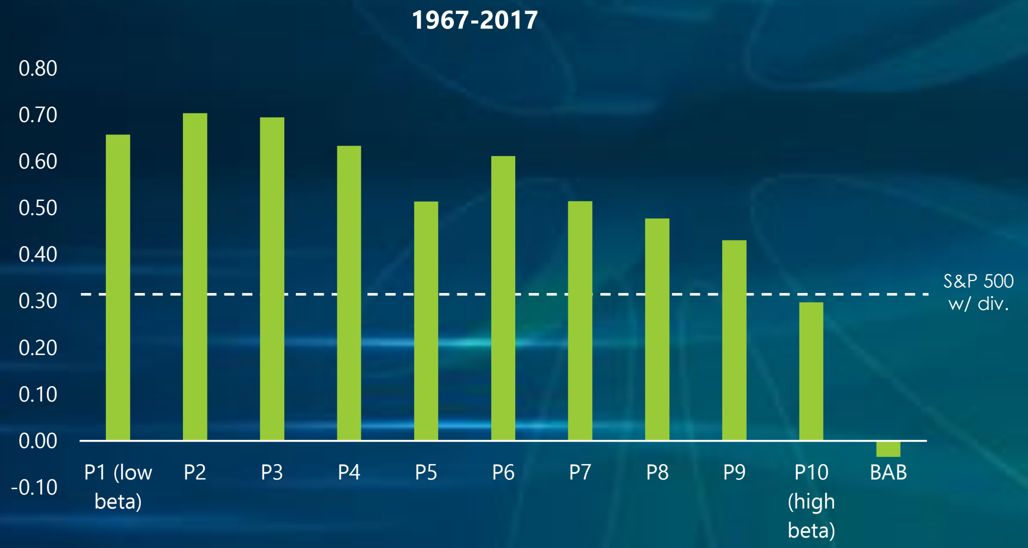
where β_i is the **estimated CAPM beta** from the 1st pass (timeseries) regression for decile portfolio i

- Alternatively, could have fitted a 2nd pass regression ala Fama-MacBeth (1973) by adding $\mathrm{Var}(\epsilon_{i,t})$ and β_i^2 from 1st pass-regression, on the RHS to account for the idiosyncratic risk and nonlinearity, but this is only for illustration purposes



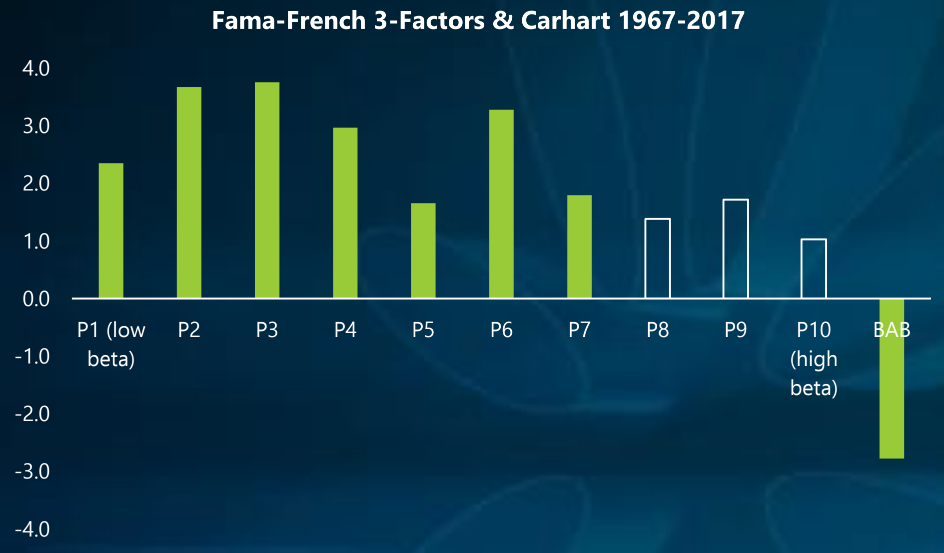
Results: Performance Evaluation

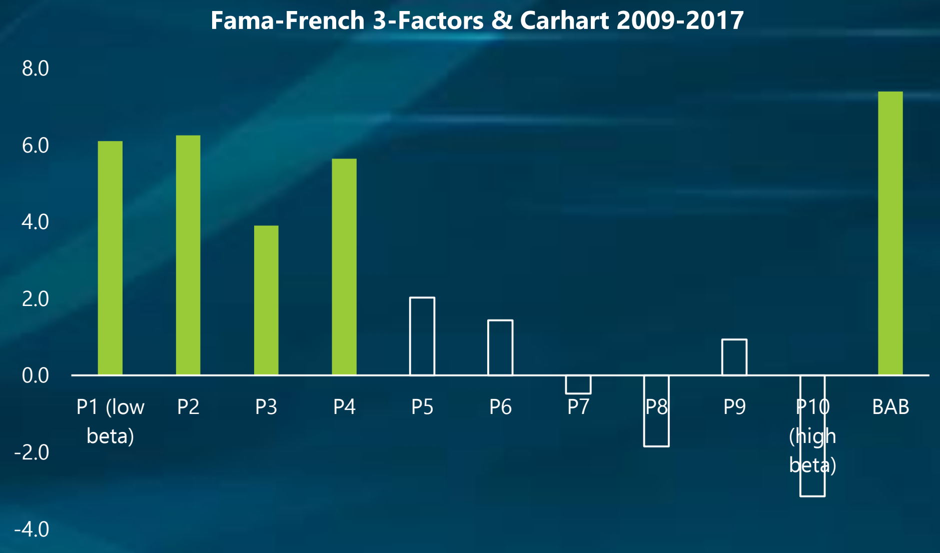
Sharpe Ratios, S&P 500 Universe

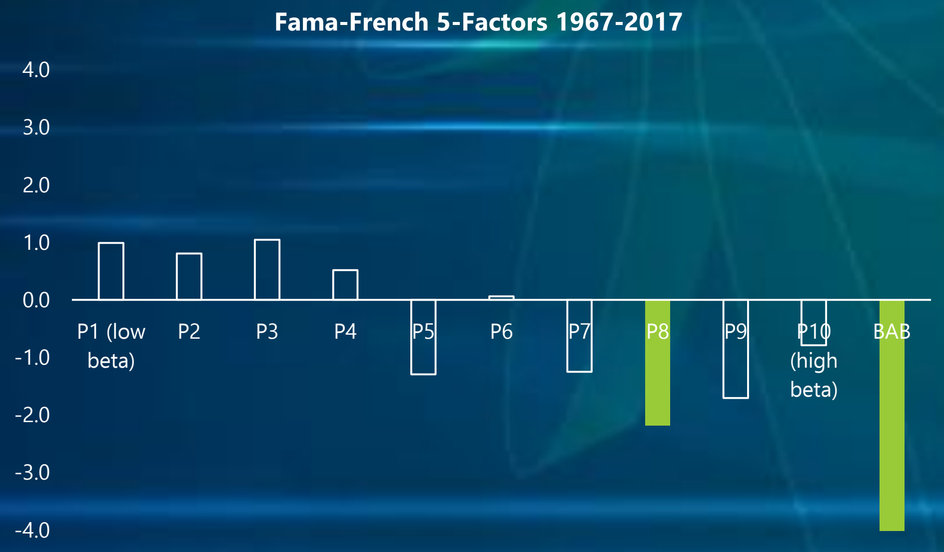


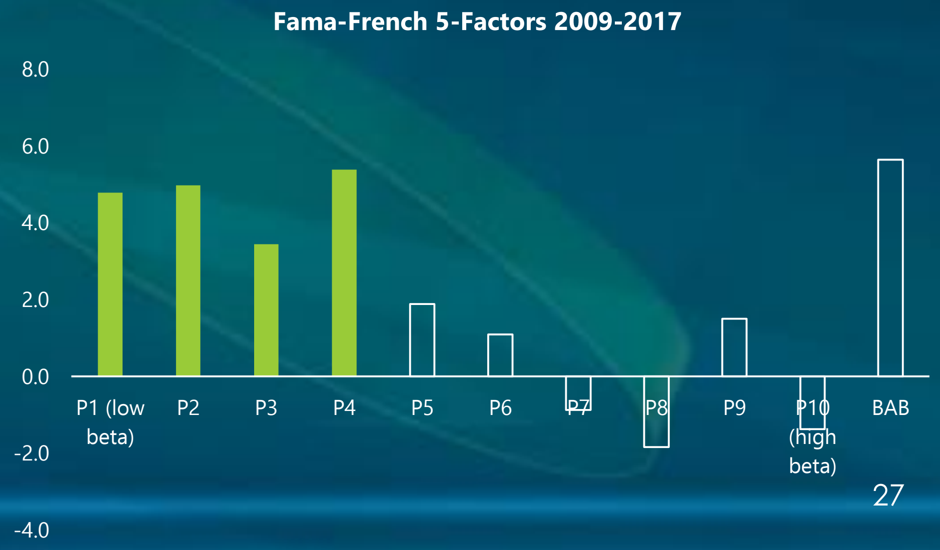
Results: Performance Evaluation

Alphas, S&P 500 Universe







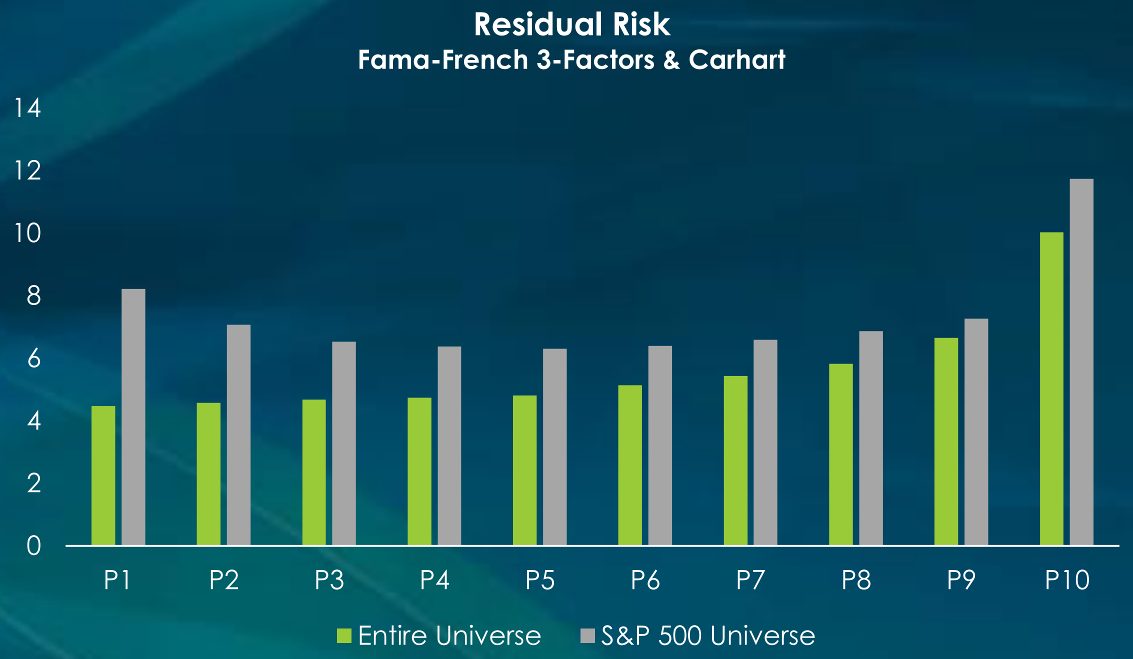


Results: Performance Evaluation
S&P 500 Universe

Portfolio	P1 (low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)	BAB	S&P 500 (w/div.)
						1967-2017						
alpha 4-factor	**2.4**	**3.7**	**3.8**	**3.0**	**1.7**	**3.3**	**1.8**	1.4	1.7	1.0	**-2.8**	
p-Value	0.07	0.00	0.00	0.00	0.08	0.00	0.07	0.18	0.14	0.53	0.07	
alpha FF5-factor	1.0	0.8	1.0	0.5	-1.3	0.1	-1.2	**-2.2**	-1.7	-0.8	**-4.0**	
p-Value	0.44	0.44	0.27	0.56	0.15	0.95	0.19	0.03	0.17	0.67	0.01	
Excess Return	8.06	9.72	10.42	9.88	8.55	10.72	9.58	9.85	9.79	8.55	-0.41	5.0
Volatility	12.26	13.83	15.01	15.61	16.64	17.54	18.65	20.65	22.74	28.77	11.83	15.1
Sharpe Ratio	0.66	0.70	0.69	0.63	0.51	0.61	0.51	0.48	0.43	0.30	-0.03	0.33
beta CAPM	0.49	0.71	0.83	0.90	0.97	1.02	1.09	1.22	1.33	1.62	-0.19	
beta 4-factor	0.59	0.78	0.88	0.95	1.01	1.07	1.12	1.24	1.31	1.47	-0.04	
						2009-2017						
alpha 4-factor	**6.1**	**6.3**	**3.9**	**5.6**	2.0	1.4	-0.5	-1.8	0.9	-3.1	**7.4**	
p-Value	0.03	0.01	0.01	0.00	0.10	0.41	0.81	0.35	0.70	0.41	0.01	
alpha FF5-factor	**4.8**	**5.0**	**3.4**	**5.4**	1.9	1.1	-0.9	-1.8	1.5	-1.4	5.6	
p-Value	0.08	0.03	0.03	0.00	0.12	0.55	0.69	0.42	0.61	0.78	0.11	
Excess Return	13.58	15.36	16.41	19.50	17.60	18.55	18.87	18.26	22.94	20.72	3.83	14.0
Volatility	9.99	10.25	11.41	12.44	14.33	16.84	20.00	20.82	24.65	32.87	13.40	13.27
Sharpe Ratio	1.36	1.50	1.44	1.57	1.23	1.10	0.94	0.88	0.93	0.63	0.29	1.06
beta CAPM	0.38	0.54	0.77	0.85	1.01	1.16	1.37	1.43	1.64	2.02	-0.48	
beta 4-factor	0.56	0.63	0.81	0.89	1.00	1.08	1.19	1.23	1.32	1.42	-0.11	

Results

Risk of decile portfolios, Entire vis-à-vis S&P 500 Universe



- S&P 500 decile portfolios have 1.1 to 1.8 times the idiosyncratic risk of the portfolios based on the entire universe of U.S. stocks
- A non-increasing residual-risk pattern across beta-sorted portfolios further strengthens the existence of the anomaly as inferred from Sharpe Ratios
- High residual variance in P9-P10 implies high S.E. and suppressed accuracy in estimated alphas

Concluding Remarks

Existence and Exploitability of the Beta anomaly

- Anomalies become weaker and arbitrage strategies unprofitable over time and once published (e.g. size effect 1980's). Beta-effect still holds?
  - Patterns of risk-adjusted returns (S.R.) on beta-sorted portfolio consistent with beta-anomaly, in both entire (1926-2017) and recent (2009-2017) periods
    - Though alpha pattern for 1926-2017 somewhat unclear, and higher beta-sorted portfolios' alphas insignificant, both due to the extreme volatility in the return timeseries of the higher beta stocks
  - Positive Sharpe ratio & alpha for BAB portfolio, in both entire and recent periods
  - Remaining alphas, after accounting for the newly introduced FF5 factor model are consistent with earlier factor models (which also fit the data better).
- Persistence of anomalous returns <u>due to</u> <u>illiquid</u> and <u>smaller stocks</u>?
  - Evidence of (1) declining risk-adjusted returns for beta-sorted portfolios and (2) a positive long-short strategy exploiting the beta-effect <u>even within the highly-liquid S&P 500 constituents</u>, though the effect is less pronounced
  - Meaningful strategies with <u>practical</u> implementation value exploiting the beta-effect are feasible: long-only portfolios or long/short (using portfolio optimization techniques); manageable number of stocks; accommodating potential limited diversification issues

‘Improving’ the beta-anomaly
Embodying a trade-direction signal generator

Idea:

- Embodying a *trade-direction signal generator* to improve gains (as measured by Sharpe Ratios and Alphas) from anomalies

  → From the pool of low-beta assets (i.e. those in P1-P3), pick the most promising

- Predicting financial market movement direction using a classification-based method, such as logistic regression, artificial neural networks (ANN) etc.

Which financial forecasting framework?

- Characteristics of financial time-series that make the use of traditional statistical techniques less attractive (Niaki and Hoseinzade, 2013):

1. non-stationarity
2. non-linear relationship between the dependent and the independent variables
3. day-to-day noisiness in the series

ANN’s can extract non-linear patterns of information hidden in the data, with high degree of accuracy

Forecasting trade-direction
The ANN Setup

Step for using classification-based method to forecast trade-direction:

- Obtain the input set of the ANN $[STE_{i,t}, AEL_{i,t};\ C_{i,t}]$ by calculating the P&L of a 3-day strategy based on a short- and a long-term price-trend variable
- Train the ANN at the end of each month
- Recursively predict P&L category for each 'future' day, using trained ANN of closest month-end
- Convert forecasted P&L category into a trade signal

Then, form an equally-weighted portfolio of all potentially profitable trades based on the ANN prediction

	P1		P2	
	Stock 1	Stock 4	Stock 2	Stock 951
20171128				
20171129				
20171130				
20171201	1	1	1	1
20171204	1	1	1	1
20171205	1	1	1	1
20171206	1	1	1	1
20171207	1	1	1	1
20171208	1	1	1	1
20171211	1	1	1	1
20171212	1	1	1	1
20171213	1	1	1	1
20171214	1	1	1	1
20171215	1	1	1	1
20171218	1	1	1	1
20171219	1	1	1	1
20171220	1	1	1	1
20171221	1	1	1	1
20171222	1	1	1	1
20171226	1	1	1	1
20171227	1	1	1	1
20171228	1	1	1	1
20171229	1	1	1	1

Forecasting trade-direction

Constructing the input set

Independent variables

- Short Term Efficiency Level (trend) $STE_{i,t} = \frac{|r_{i,t}^K|}{|\sum_{d=1}^K r_{i,t}^d|}$
- Average Efficiency Level (average trend) $AEL_{i,t} = \frac{1}{L}\sum_{l=0}^{L-1} STE_{i,t-l}$

where $r_{i,t}^K = \sum_{k=0}^{K-1} \ln(1 + r_{i,t-k})$

Dependent variable

- 3-day Trading Strategy for calculating $P\&L_{i,t}$

Trade stock i at day t with direction $D_{i,t}$ (i.e. long if 1; short if -1) and exit the trade at $t+j$ with $j \geq 1$ (i.e. position is held for at least one day) when either of the following occurs:

1. $STE_{i,t+j} < STE_{i,t}$
2. $AEL_{i,t+j} < AEL_{i,t}$
3. $j = 3$

where $D_{i,t} = \begin{cases} 1 \ \ if \ \ r_{i,t}^K \geq 0 \\ -1 \ \ \text{otherwise} \end{cases}$

- The computed $P\&L$ of the 3-day strategy based on STE, AEL and D is then classified in one of the nine-categories, to create the dependent variable $C_{i,t}$. The 9-classes (rather than a standard 3-classes) setup allows for the identification and treatment of outliers

Three-day Trade Strategy P&L Categories

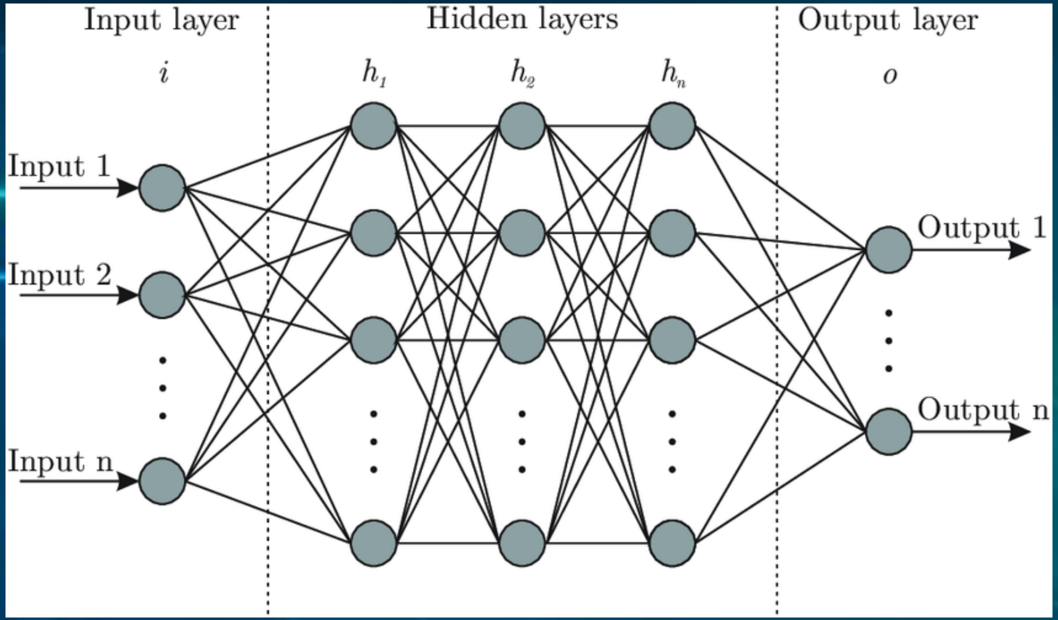
P&L Range	P&L Range for w=0.3%	P&L Class (ANN Input)
-∞ < P&L ≤ -7w	-∞ < P&L ≤ -2.10%	-1
-7w < P&L ≤ -5w	-2.10% < P&L ≤ -1.50%	-0.75
-5w < P&L ≤ -3w	-1.50% < P&L ≤ -0.90%	-0.5
-3w < P&L ≤ -w	-0.90% < P&L ≤ -0.30%	-0.25
-w < P&L ≤ w	-0.30% < P&L ≤ 0.30%	0
w < P&L ≤ 3w	0.30% < P&L ≤ 0.90%	0.25
3w < P&L ≤ 5w	0.90% < P&L ≤ 1.50%	0.5
5w < P&L ≤ 7w	1.50% < P&L ≤ 2.10%	0.75
7w < P&L ≤ +∞	2.10% < P&L ≤ +∞	1

Forecasting trade-direction

Embodying a trade-direction signal generator

ANN Architecture and Training:

- Two-layer (input-layer & one hidden-layer) feedforward neural network, with 20 neurons in the hidden layer
- Trained based on 750 obs of the input set $[STE_{i,t}, AEL_{i,t};\ C_{i,t}]$, for $K = 10$ and $L = 20$
- Used 80% of sample for my *training set* to teach the network, and the remaining as a *validation set* for improving the network's accuracy



Forecasting trade-direction

Embodying a trade-direction signal generator

- The network is trained at each month-end $t = T$ for every stock i allocated in decile-portfolios P1-P3
- The P&L category $\hat{C}_{i,t}$ is then forecasted using the trained ANN, recursively for <u>each day</u> in the following month
- The ANN output $\hat{Z}_{i,t}$ is converted to P&L category as: $\hat{C}_{i,t} = \max(-1, \min(1, 0.25 \cdot round(4 * \hat{Z}_{i,t})))$
- A 'buy signal' for day $T+1$, $\hat{S}_{i,T}$ is inferred when $0.25 \leq D_{i,T} \cdot \hat{C}_{i,T} \leq 0.75$. In example:
  - $D_{i,T} = 1$ and predicted P&L>0 → strategy indicates long and it is forecasted to be correct
  - $D_{i,T} = -1$ and predicted P&L<0 → strategy indicates short and it is forecasted to be incorrect

Three-day Trade Strategy P&L Categories

P&L Range	P&L Range for w=0.3%	P&L Class (ANN Input)	
-∞ < P&L ≤ -7w	-∞ < P&L ≤ -2.10%	-1	→ outliers
-7w < P&L ≤ -5w	-2.10% < P&L ≤ -1.50%	-0.75	if D=1, strategy indicated long → should have sold short
-5w < P&L ≤ -3w	-1.50% < P&L ≤ -0.90%	-0.5	**if D=-1, strategy indicated short → should have gone long**
-3w < P&L ≤ -w	-0.90% < P&L ≤ -0.30%	-0.25	
-w < P&L ≤ w	-0.30% < P&L ≤ 0.30%	0	→ Unclear singal → 'No trade'
w < P&L ≤ 3w	0.30% < P&L ≤ 0.90%	0.25	**if D=1, strategy indicated long → P&L and D in consonance**
3w < P&L ≤ 5w	0.90% < P&L ≤ 1.50%	0.5	if D=-1, strategy indicated short → P&L and D in consonance
5w < P&L ≤ 7w	1.50% < P&L ≤ 2.10%	0.75	
7w < P&L ≤ +∞	2.10% < P&L ≤ +∞	1	→ outliers

To compute $\hat{Z}_{i,T}$, $\hat{C}_{i,T}$ and then $\hat{S}_{i,T}$, we need daily returns up to date T
Specifically, a total of $K+L-1$ daily returns i.e. the returns from $T-K-L+2$ to T

‘Improving’ the beta-anomaly

Descriptive Statistics of ANN Portfolios

Number of Equities	Universe	
	Entire	S&P 500
Classified in P1-P3	966	106
w\ ANN Signaling a Long Position	179	17
Daily %	17.29%	15.62%

Among all the low-beta equities assigned to decile-portfolios P1 to P3, on average 15 to 17% are picked up by our ANN as potentially profitable opportunities

Results: Performance Evaluation

ANN portfolios, Both Universes

Universe	Entire					S&P 500				
Portfolio	**P1-P3**	**P1**	**P2**	**P3**	**S&P 500 (w/div.)**	**P1-P3**	**P1**	**P2**	**P3**	**S&P 500 (w/div.)**
	1930-2017					**1967-2017**				
alpha 4-factor	**54.7**	**69.6**	**44.4**	**39.6**		**6.4**	**4.8**	**6.9**	**6.1**	
p-Value	0.00	0.00	0.00	0.00		0.00	0.04	0.00	0.00	
alpha FF5-factor	**62.5**	**80.3**	**45.0**	**40.6**		**3.3**	2.9	**3.7**	**3.3**	
p-Value	0.00	0.00	0.00	0.00		0.01	0.21	0.05	0.07	
Excess Return	57.2	70.4	45.9	43.5	5.8	11.4	8.4	11.2	11.8	5.0
Volatility	24.1	29.6	25.8	23.7	18.8	14.7	17.5	16.7	18.2	15.1
Sharpe Ratio	2.37	2.38	1.78	1.84	0.31	0.77	0.48	0.67	0.65	0.33
Relative Sharpe	7.7	7.7	5.8	6.0	1	2.3	1.5	2.0	2.0	1
beta CAPM	0.91	0.91	0.94	0.98		0.71	0.55	0.69	0.85	
beta 4-factor	0.78	0.78	0.88	0.82		0.76	0.61	0.75	0.89	
					2009-2017					
alpha 4-factor	**48.5**	**73.2**	**25.9**	**19.0**		**4.4**	**8.5**	6.2	1.9	
p-Value	0.00	0.00	0.00	0.00		0.07	0.05	0.14	0.53	
alpha FF5-factor	**48.9**	**73.7**	**26.2**	**19.4**		3.9	5.8	6.3	2.0	
p-Value	0.00	0.00	0.00	0.00		0.11	0.18	0.14	0.52	
Excess Return	58.40	80.16	37.02	32.14	14.0	14.36	14.20	14.85	15.56	14.0
Volatility	15.14	16.07	15.49	17.57	13.3	10.78	11.98	12.26	14.17	13.3
Sharpe Ratio	3.86	4.99	2.39	1.83	1.06	1.33	1.19	1.21	1.10	1.06
Relative Sharpe	3.7	4.7	2.3	1.7	1	1.3	1.1	1.1	1.0	1
beta CAPM	0.90	0.80	0.95	1.09		0.61	0.40	0.47	0.83	
beta 4-factor	0.68	0.56	0.75	0.85		0.71	0.55	0.54	0.93	

Appendix

Pre-ranking Betas Robustness

- Tested the following specifications/stock-inclusion rules for estimating ex-ante betas:
  - Correlations and Volatilities estimated separately (method A)
    - Method A1:

      Required 1260 (5-year) consecutive non-missing obs from month-end
      Estimated correlations on 1260 obs and volatilities on 252 obs
    - Method A2:

      Required at least 750 (3-year) consecutive non-missing obs from month-end
      Estimated correlations on 1260-750 obs and volatilities on 252 obs
    - Method A3:

      Required at least 750 (3-year) non-missing obs in the past 1260 days from month-end
      and at least 120 (6-month) non-missing obs in the past 252 days from month-end
      Estimated correlations on 1260-750 obs and volatilities on 252-120 obs
  - Standard CAPM model (method B):
    Based on 1-day log-returns of 252-days, requiring at least 120 non-missing returns

Dropping the consecutive obs requirement

Possible Sources of Deviation

- Treatment of NASDAQ securities → Inclusion of NASDAQ stocks with prices based on bid/ask spread average
  - NASDAQ securities were not required to report transactions prior 1992
  - Reported CRSP prices are bid/ask spread averages rather than actual trading prices
  - 'Returns on bid/ask averages have <u>different characteristics</u> from actual trade-based returns'

    Data Descriptions Guide, CRSP US Stock & US Index Databases, June 2018
- Treatment of penny stocks → Inclusion of penny stocks
- Pre-ranking Beta Calculation
  - Recall: **3-day** log returns for **correlations** and **1-day** log-returns for **st. devs**; $t \equiv$ Month-end

    Method 1: Correlations: $\sum_{k=0}^{2} \ln(1 + r_{i,t+k})$; St. Devs: $\ln(1 + r_{i,t})$

    Method 2: Correlations: $\sum_{k=0}^{2} \ln(1 + r_{i,t+k})$; St. Devs: $\ln(1 + r_{i,t-2})$

  → Method 2 the logical choice because no need for 'future' data (i.e. ahead of respective month-end)
- NYSE Beta Breakpoints
  - NYSE membership based on month-end (t) or 2 days before $(t-2)$? → NYSE membership defined by EXCHCD value at day t
- Treatment of stocks with ex-ante betas equal to deciles
  - e.g. Stock i with $\hat{\beta}_i = D1$ enters decile portfolio $\mathrm{P1}$ or $\mathrm{P2}$? → Stock i classified in $\mathrm{P1}$
- Treatment of stocks whose beta calculated at month-end, but stop trading <u>within next</u> month

    Method 1: Select stocks that trade throughout the entire month, and rebalance monthly

    Method 2: Rebalance within month

  → Implemented both (reported the former)

Possible Sources of Deviation

- Month-end Beta Deciles
  - Using NYSE equities
  - Using equities from all exchanges
  - → Employed NYSE Breakpoints

- Literature constructs the long/short pricing factor to measure the beta-effect based on different quantiles
  - Auer and Schuchmacher (2015) long the 1st quartile and short the 4th, excluding all the stocks in between

Sanity Check

Correlations with CRSP's Beta Decile Portfolios

- CRSP's Beta Decile Portfolios
  - Betas are estimated annually using the past 252 observations (with at least 126 non-missing obs).
  - Betas are 'Scholes and Williams (1977) betas' to control for nonsynchronous trading.
  - Includes all types of securities (i.e. all SHRCD's) that trade in NYSE/NYSE MKT(AMEX) only.
  - Excludes securities whose returns are based on bid/ask spread average rather than actual price.

Universe	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
Entire	0.61	0.78	0.86	0.89	0.90	0.92	0.93	0.94	0.94	0.91
S&P 500	0.44	0.71	0.80	0.84	0.87	0.88	0.90	0.91	0.91	0.88

Selected References

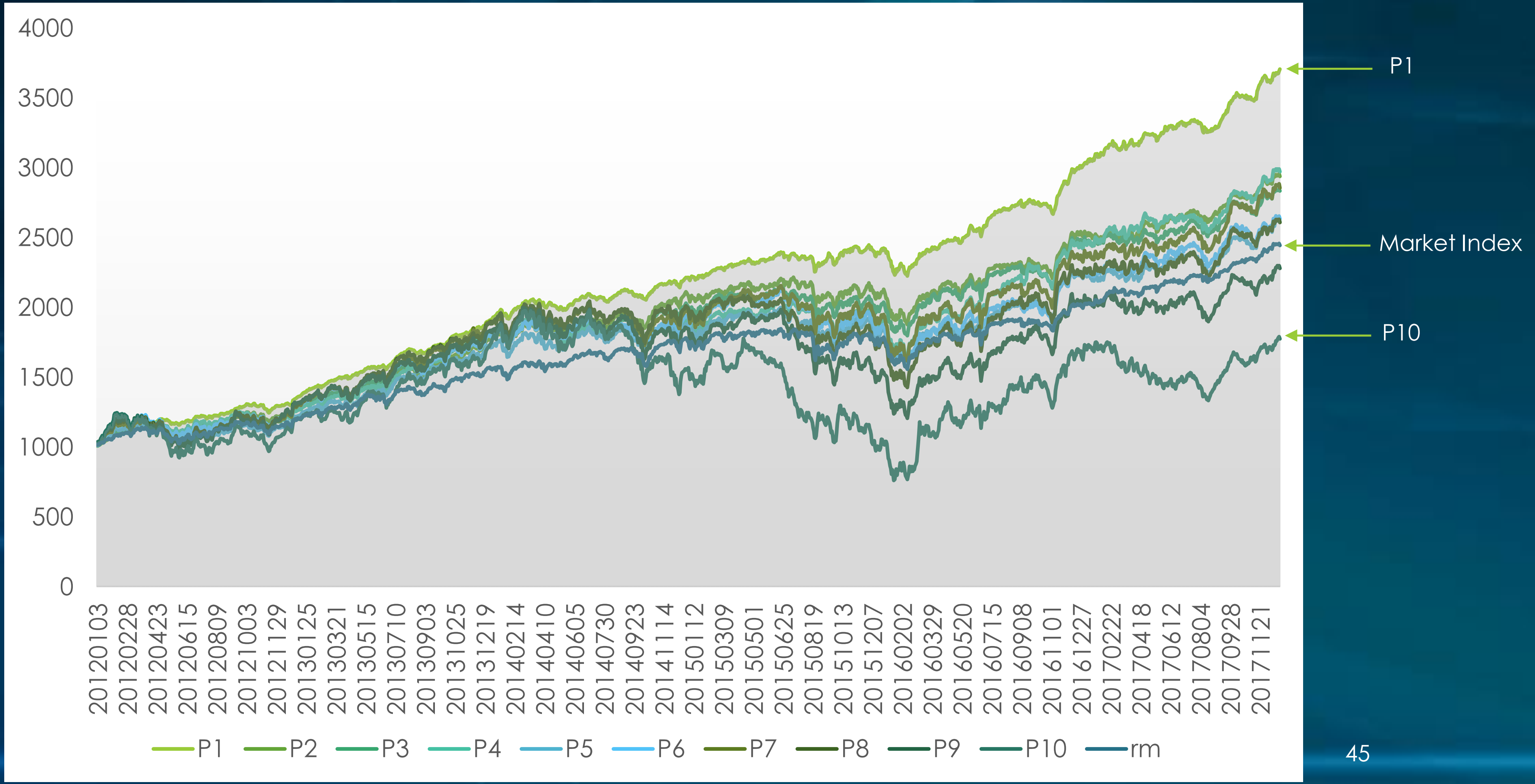
- Auer B. R., and F. Schuhmacher, (2015), “Liquid Betting against Beta in Dow Jones Industrial Average Stocks”, *Financial Analysts Journal* 71(6): 30-43.
- Black F., Jensen M. C., Scholes M., (1972), The Capital Asset Pricing Model: Some Empirical Tests. In: Jensen, M.C.(Ed.), Studies in the Theory of Capital Markets, Praeger New York, pp. 79–121.
- Bodie, Z., A. Kane, and A. Marcus, (2014), “Investments”, 10th ed. Columbus, McGraw-Hill Education.
- Carhart, M. (1997), "On persistence in mutual fund performance", *Journal of Finance* 52, 57-82.
- Fama, E.F. and French, K.R. (1993), "Common risk factors in the returns on stocks and bonds", *Journal of Financial Economics* 33, 3-56.
- Fama E.F., French, K. R. (2015), "A five-factor asset pricing model", Journal of Financial Economics 116(1): 1-22.
- Frazzini A., and L. H. Pedersen, (2014), ‘Betting Against Beta’, *Journal of Financial Economics* 111: 1-25.
- Gromb D., and Vayanos D, (2010), ‘Limits of Arbitrage’, *Annual Review of Financial Economics* 2:1, 251-275.
- Li X., R. Sullivan, and L. Garcia-Feijóo, (2014), “The Limits to Arbitrage and the Low-Volatility Anomaly.” *Financial Analysts Journal* 70(1): 52–63.
- Li X., R. Sullivan, and L. Garcia-Feijóo, (2016), “The Low-Volatility Anomaly: Market Evidence on Systematic Risk vs. Mispricing.” *Financial Analysts Journal* 72(1): 36–47.

Selected References

- Niaki S. T. A., and Hoseinzade S. (2013) "Forecasting S&P 500 Index Using Artificial Neural Networks and Design of Experiments", Springer Journal of Industrial Engineering International, pp. 1-9.
- Pastor L., and Stambaugh R. (2003), “Liquidity risk and expected stock returns”, Journal of Political Economics, 111, pp. 642-685.
- Sethi M., Treleaven P., and Del Bano Rollin S. (2014) "Beating The S&P 500 Index - A Successful Neural Network Approach", International Joint Conference on Neural Networks (IJCNN), July 2014.
- Vasicek, O. A. (1973), “A Note on using Cross-sectional Information in Bayesian Estimation on Security Beta’s,” The Journal of Finance, 28(5), 1233–1239.

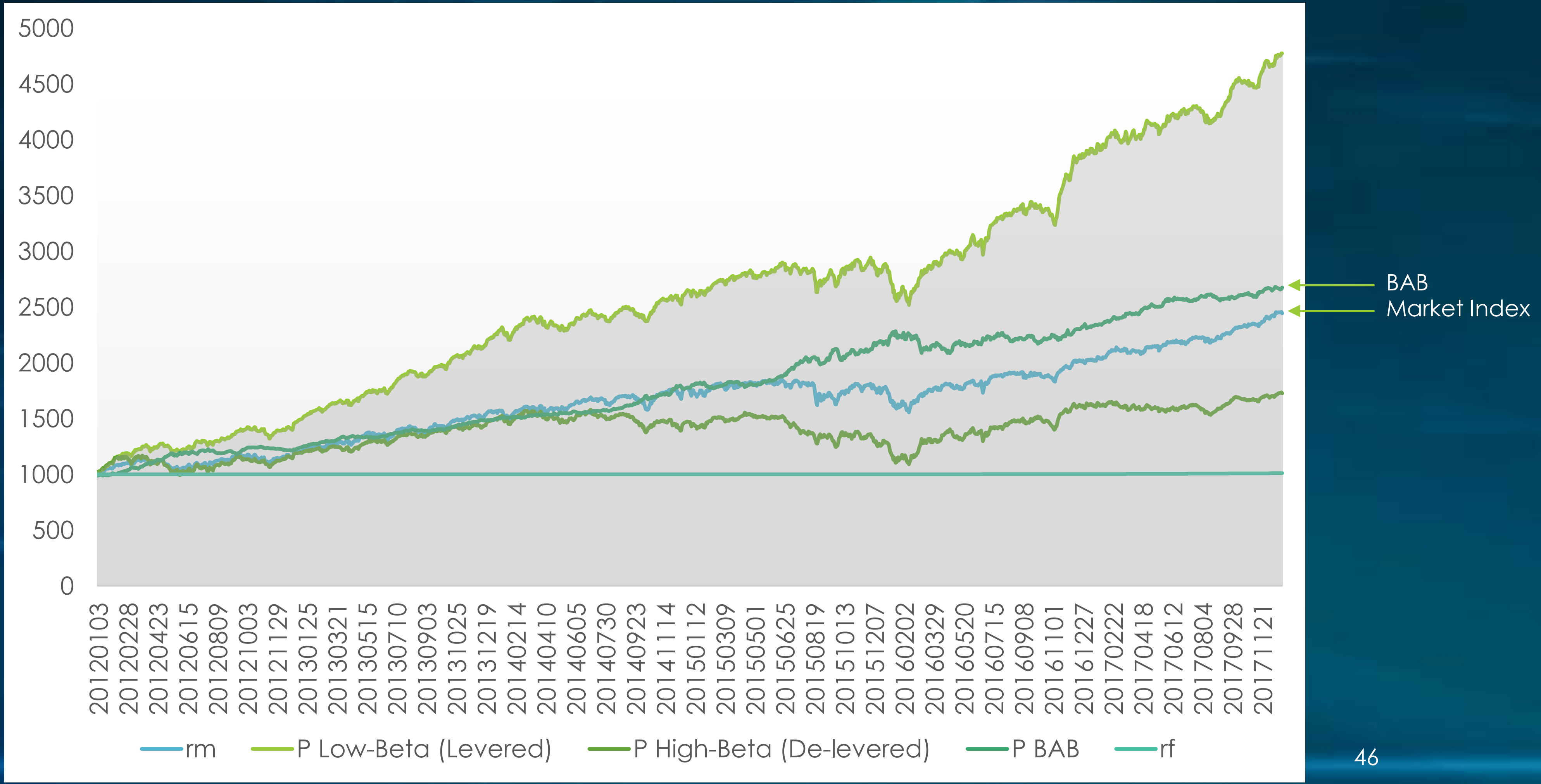
Constructed Portfolios

Beta-Decile Portfolios: $1000 since February 2012, Entire Universe



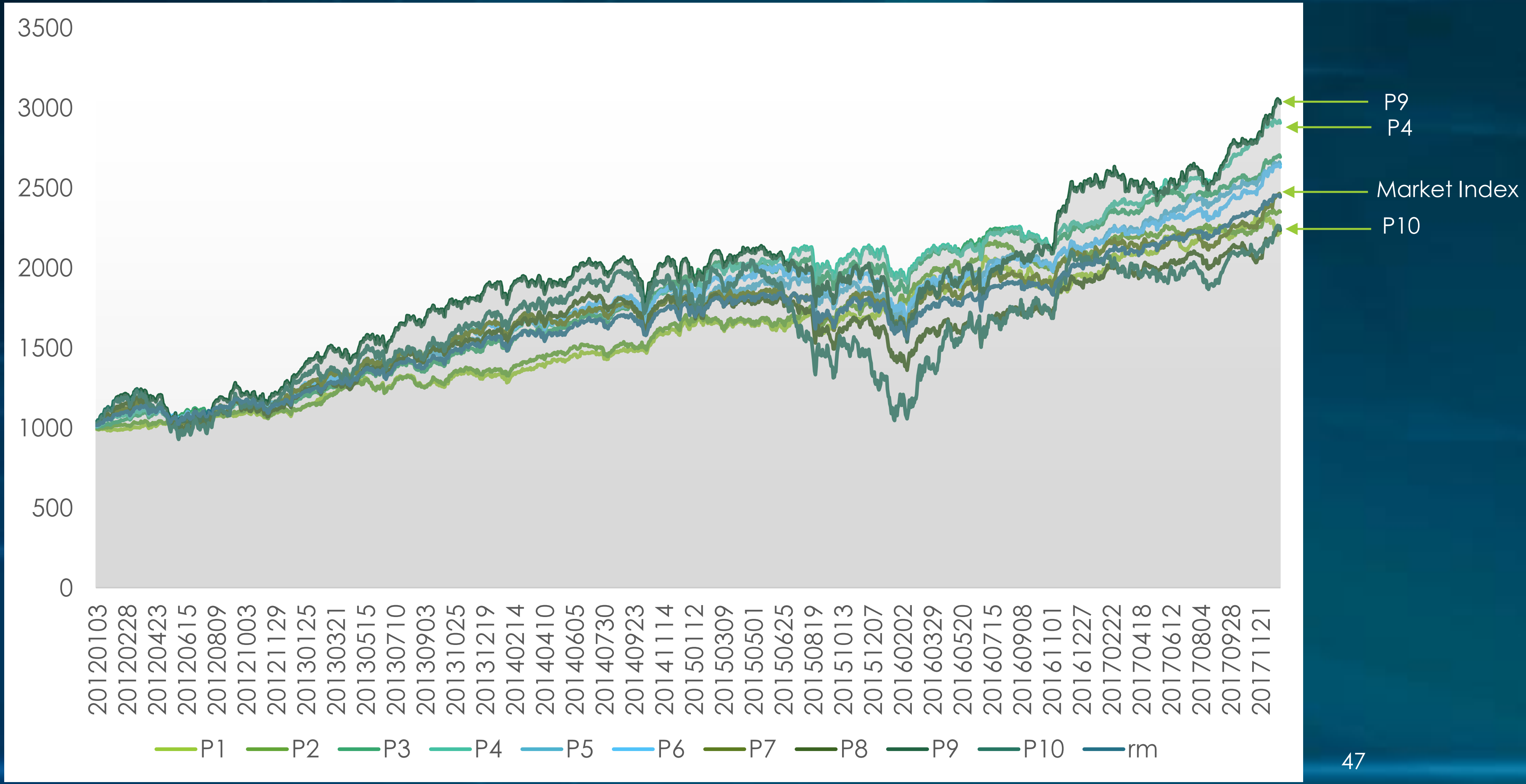
Constructed Portfolios

BAB & Low/High Beta Portfolios: $1000 since February 2012, Entire Universe



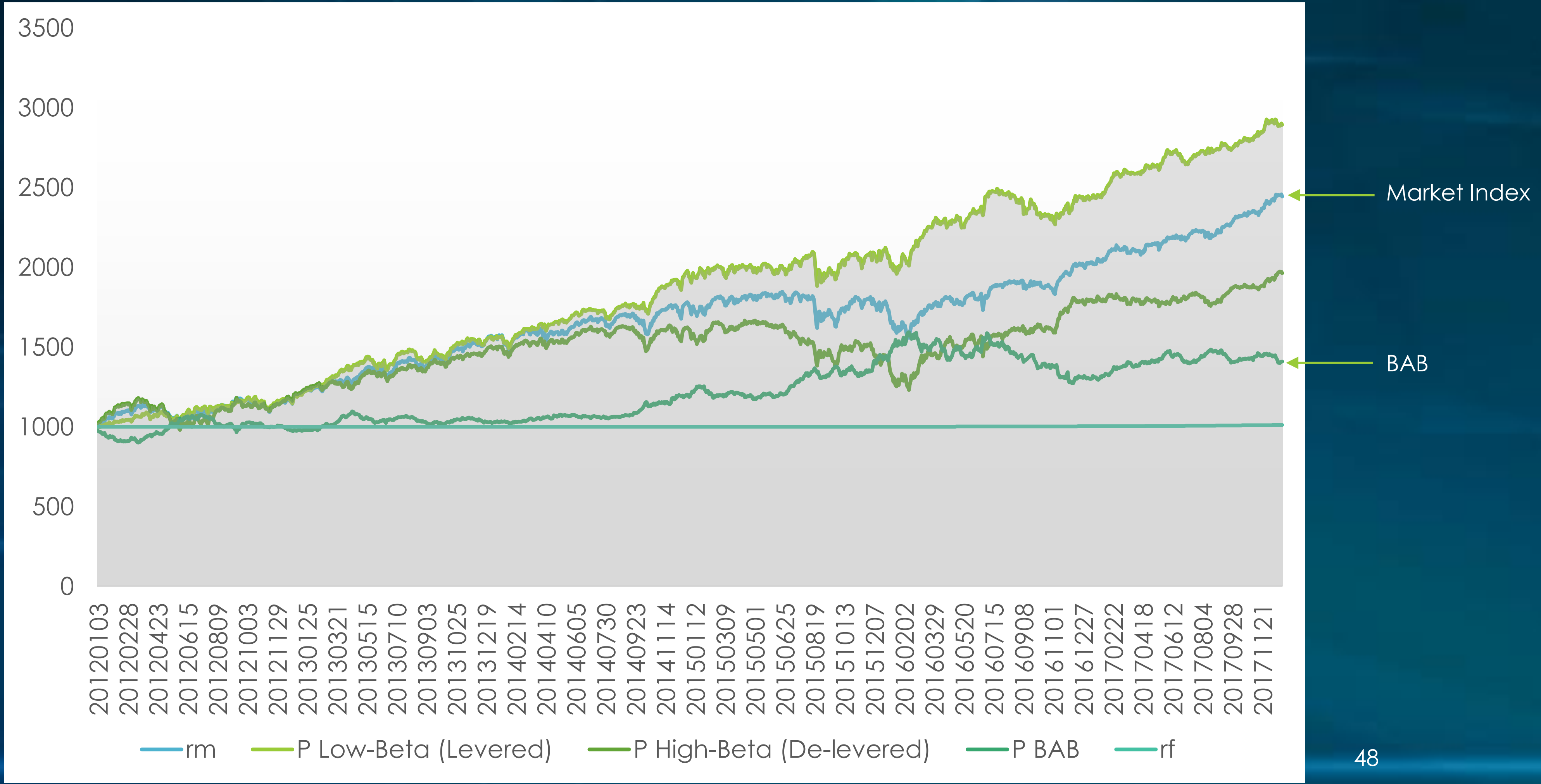
Constructed Portfolios

Beta-Decile Portfolios: $1000 since February 2012, S&P 500 Universe



Constructed Portfolios

BAB & Low/High Beta Portfolios: $1000 since February 2012, S&P 500 Universe



Literature on 'The Limits of Arbitrage'

- **Questions to address:**
  - *Why anomalies arise?*
    Rationalize demand shocks, deviating from the conventional view that shocks that push prices away of fundamentals are attributed to investors' cognitive biases
  - *Why arbitrage fails to eliminate them?*
    Limited capital provision (leverage or equity capital constraints) and other costs (such as the costs associated with establishing and financing long and short positions) faced by *arbitrageurs* in the process of eliminating mispricings
  - *Are arbitrageur (risk-taking) decisions socially optimal?*
- **Distinguish between investor types. Agent types**:
  - Arbitrageurs: hedge funds, trading desks, and mutual funds → trade on the mispricing (i.e. they buy the underpriced & short the overpriced)
  - Outside investors: less sophisticated investors
- **Model Phenomena such as**:
  - Law of One Price violations: assets of similar payoffs trading at significantly different prices
  - Efficient Market Hypothesis violations: asset returns are predictable
  - Contagion: shock transmission across segmented markets
  - Amplification: small shocks resulting in large and prolonged price drops
- **Other Considerations:**
  - Develop models that fully endogenize financial constraints, by deriving such constraints from an optimal financial contracting framework
    e.g. leverage constraints and equity capital constraints, stem from a common friction, i.e. provision of limited capital (capital providers in the case of leverage constraints are the lenders in repo markets, and in the equity capital constraints case the outside investors who hold positions in mutual funds) but the majority of models in the literature considers them separately
  - Develop models to facilitate policy analysis by deviating from Pareto optimality in equilibrium, making public intervention meaningful

Exploiting the beta-anomaly
Optimizing the Mispriced Portfolios

Mixing the mispriced portfolios with a passive investment on a broad market index, to accommodate diversification concerns by means of the Treynor-Black (1973) portfolio optimization technique. How?

- **The Mixed Portfolio** return is then:

$$r_M = w_A^* r_A + (1 - w_A^*) r_{ETF}$$

with the optimal weight for the active portfolio given by

$w_A^* = \frac{w_A}{1+(1-\beta_A)\, w_A}$ with $w_A = \frac{\alpha_A / Var(e_A)}{E(R_{ETF}) / Var(R_{ETF})}$

where $\alpha_A = \sum_{i=1}^{n} w_i^{*N} \alpha_i$ and $\beta_A = \sum_{i=1}^{n} w_i^{*N} \beta_i$, and $Var(e_A) = \sum_{i=1}^{n} \left(w_i^{*N}\right)^2 Var(e_i)$.

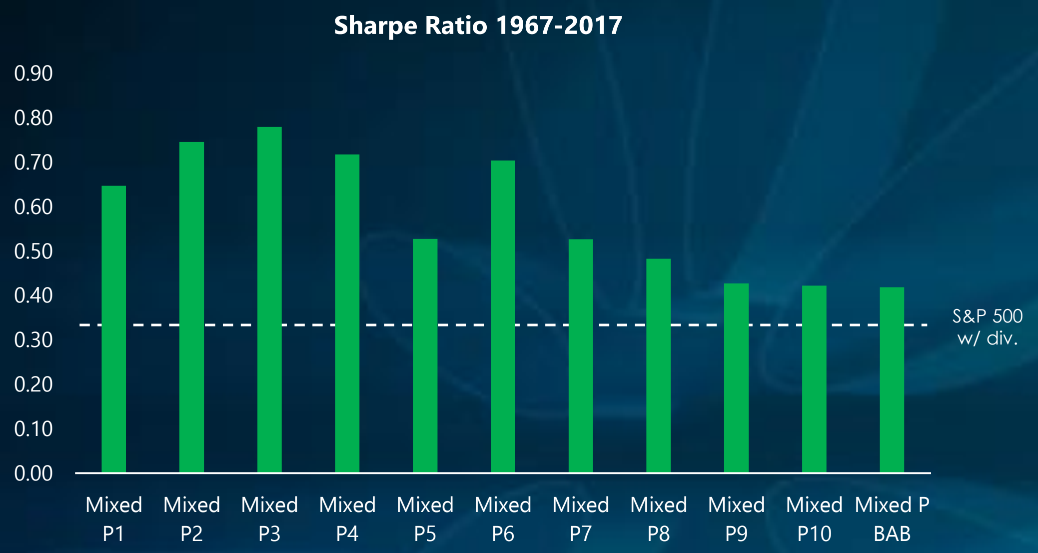
- Multiple mispriced securities can be combined into an **Active Portfolio** with return $r_A = \sum_{i=1}^{n} w_i^{*N} r_i$

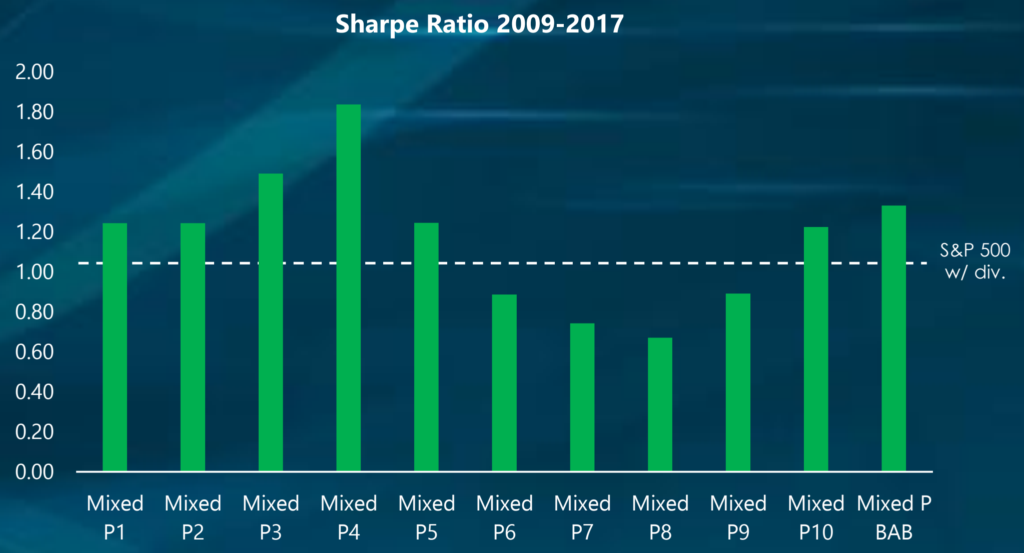
The optimal weight of each mispriced security in the active portfolio is obtained by:

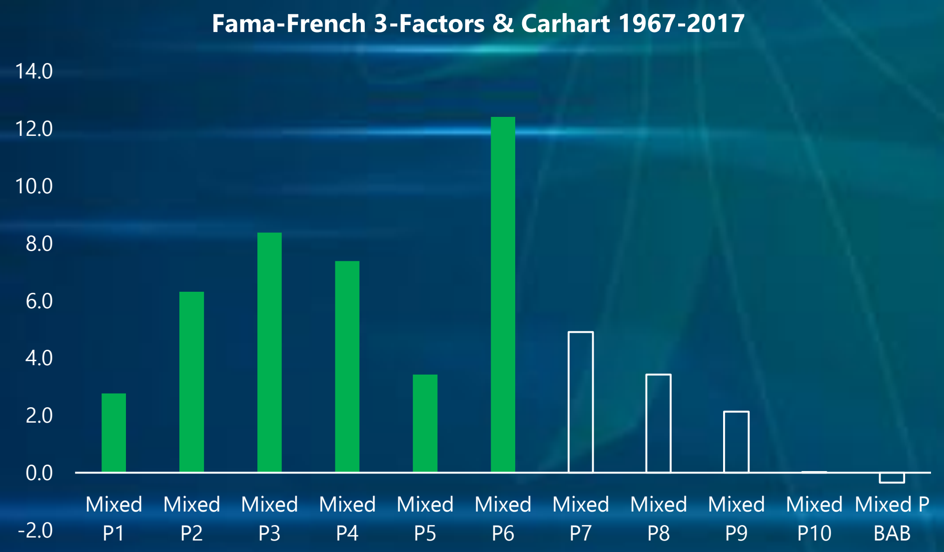
$w_i^{*N} = \frac{\frac{\alpha_i}{Var(e_i)}}{\sum_{i=1}^{n} \frac{\alpha_i}{Var(e_i)}}$ with $\sum_{i=1}^{n} w_i^{*N} = 1$

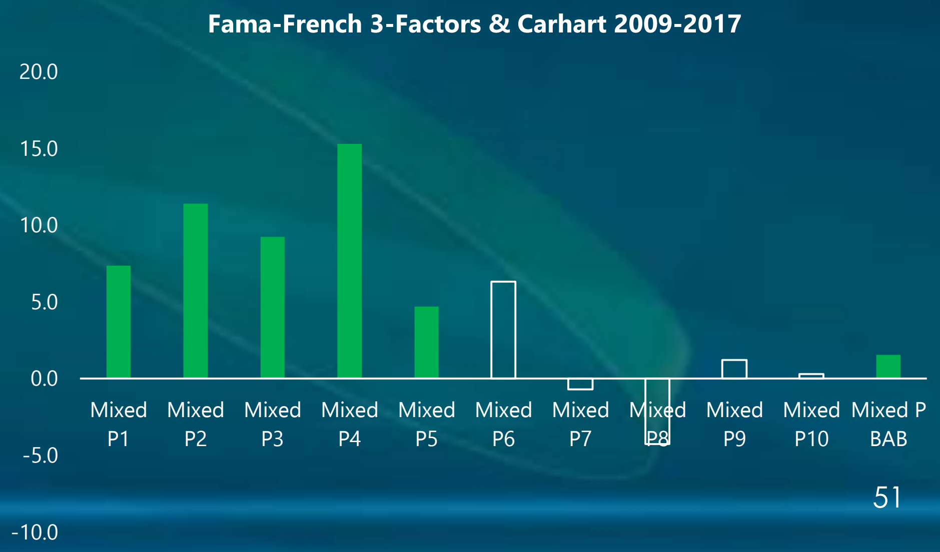
Results: Practical Exploitability

Risk-adjusted returns, Mixed portfolios I, S&P 500 Universe









Results: Practical Exploitability

Mixed portfolios I, S&P 500 Universe

Portfolio	P1 (low beta)	P2	P3	P4	P5	P6	P7	P8	P9	P10 (high beta)	BAB
1967-2017											
alpha 4-factor	**2.8**	**6.3**	**8.4**	**7.4**	**3.4**	**12.4**	**4.9**	3.4	2.1	0.0	-0.3
p-Value	0.08	0.00	0.00	0.00	0.10	0.00	0.07	0.18	0.14	0.95	0.33
alpha FF5-factor	1.2	1.6	2.8	1.8	-2.5	1.3	-2.8	**-4.6**	-2.0	-0.2	**-1.0**
p-Value	0.42	0.40	0.21	0.45	0.21	0.71	0.30	0.07	0.19	0.58	0.00
Excess Return	8.43	12.50	16.08	15.96	11.56	24.60	16.08	15.59	10.73	5.73	4.83
Volatility	13.03	16.77	20.63	22.25	21.92	34.95	30.55	32.31	25.14	13.57	11.54
Sharpe Ratio	0.65	0.75	0.78	0.72	0.53	0.70	0.53	0.48	0.43	0.42	0.42
Relative Sharpe	2.0	2.3	2.4	2.2	1.6	2.1	1.6	1.5	1.3	1.3	1.3
beta CAPM	0.40	0.52	0.67	0.80	0.98	1.21	1.33	1.60	1.43	0.83	0.71
beta 4-factor	0.51	0.61	0.73	0.86	1.02	1.31	1.34	1.60	1.38	0.91	0.78
2009-2017											
alpha 4-factor	**7.4**	**11.4**	**9.2**	**15.3**	**4.7**	6.3	-0.7	-4.3	1.2	0.3	**1.5**
p-Value	0.02	0.01	0.01	0.00	0.09	0.37	0.90	0.40	0.70	0.72	0.04
alpha FF5-factor	**5.8**	**9.2**	**8.3**	**14.8**	**4.5**	5.3	-1.8	-4.2	1.9	-0.2	**1.1**
p-Value	0.08	0.03	0.02	0.00	0.10	0.48	0.78	0.48	0.61	0.86	0.19
Excess Return	13.31	15.70	18.20	26.98	20.77	29.57	26.25	23.12	24.92	13.74	12.70
Volatility	10.70	12.62	12.20	14.68	16.68	33.35	35.36	34.49	27.93	11.22	9.53
Sharpe Ratio	1.24	1.24	1.49	1.84	1.25	0.89	0.74	0.67	0.89	1.22	1.33
Relative Sharpe	1.2	1.2	1.4	1.7	1.2	0.8	0.7	0.6	0.8	1.2	1.3
beta CAPM	0.26	0.20	0.50	0.66	1.07	1.79	2.12	2.12	1.82	0.76	0.66
beta 4-factor	0.47	0.34	0.56	0.70	1.01	1.31	1.54	1.58	1.40	0.92	0.76

Results: Practical Exploitability

Mixed portfolios II, Both Universes

Universe	Entire	S&P 500
	1930-2017	**1967-2017**
alpha 4-factor	**16.1**	**6.2**
p-Value	0.00	0.06
alpha FF5-factor	**16.2**	4.5
p-Value	0.00	0.18
Excess Return	20.0	6.5
Volatility	16.6	28.3
Sharpe Ratio	1.21	0.23
Relative Sharpe	3.9	0.7
beta CAPM	0.57	0.53
beta 4-factor	0.40	0.53
	2009-2017	
alpha 4-factor	**33.0**	**14.6**
p-Value	0.00	0.01
alpha FF5-factor	**34.4**	**12.0**
p-Value	0.00	0.02
Excess Return	37.16	13.23
Volatility	18.95	15.17
Sharpe Ratio	1.96	0.87
Relative Sharpe	1.9	0.8
beta CAPM	0.64	-0.18
beta 4-factor	0.18	0.05